Finite Element Implementation and Experimental Validation of 2D/3D Magnetic Force formulae

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A general framework for the computation of both the local and the global electromagnetic forces/torques is presented. This framework relies on a unique principle applicable in 2D and 3D problems, in the presence orbitrary linear and nonlinear materials. An experimental validation setup is also described in details, for which accurate measurements have been carried out, so that it can serve as a benchmark. Issues regarding accuracy and convergence are also dealt with.

***Index Terms*—Magnetic Force, Finite Element Analysis, Permanent Magnet.**

I. Introduction

Practical methods to compute electromagnetic forces are often presented as a set of apparently unrelated formulae applicable in well-defined conditions (e.g. formulae for the torque acting on the rigid rotor of an electrical machine, formulae for the force acting on pointwise particles like charges, dipoles and moments, formulae for the nodal forces in finite element meshes, . . .) Besides the work of Alain Bossavit (e.g. [1]) much less is said however in the literature about the common theoretical bakground behind that wealth of disparate formulae. The first purpose of this paper is to recall how such a background can be established from first thermodynamic principles and, then, to deduce from it a robust and generic pratical finite element (FE) implementation for arbitrary materials and 2D/3D models.

The second purpose of the paper is to describe a benchmark application in details, for which accurate measurements have been carried out. The setup of this benchmark consists of two cylindrical co-axial PM barrels with a variable separating distance. Extensive geometrical and material data are given in the paper, so that it can serve as benchmark for the validation of FE implementations of magnetic force formulae. Finally, a number of issues regarding the accuracy of the computed forces and the convergence when the mesh is refined are dealt with.

II. Magnetic forces: a definition

Only magnetic forces are dealt with in this paper. Forces of electric origin would be defined the same way, considering the electric flux densisty $\mathbf{d}$ instead of the magnetic flux density $\mathbf{b}$. Let for instance (1), with μ a constant, be the amount of energy enclosed in an infinitesimal volume element $\Delta\Omega$, and $\mathbf{v}$ be the velocity field that represents a continuous deformation (possibly a virtual one, in the sense of the virtual principle method) of the system.

$$\Delta\Psi(\mathbf{b}) = \frac{\mathbf{b}\cdot\mathbf{b}}{2\mu}\Delta\Omega. \tag{1}$$

The variation of energy under this deformation is expressed by evaluating the Lie derivative $L_\mathbf{v}$ of (1)

$$\begin{aligned} L_\mathbf{v}\left\{\Delta\Psi(\mathbf{b})\right\} &= \partial_\mathbf{b}\left\{\Delta\Psi(\mathbf{b})\right\}\cdot L_\mathbf{v}\mathbf{b} + \left\{L_\mathbf{v}\Delta\Psi\right\}(\mathbf{b}) \\ &= \left\{\mathbf{h}(\mathbf{b})\cdot L_\mathbf{v}\mathbf{b} + \sigma_{EM} : \operatorname{grad}\mathbf{v}\right\}\Delta\Omega. \end{aligned} \tag{2}$$

The first term accounts for the dependency in $\mathbf{b}$ of the function $\Delta\Psi$, and it represents the rate of magnetic work, i.e. the variation of magnetic stored energy, since one has

$$\partial_\mathbf{b}\left\{\Delta\Psi(\mathbf{b})\right\} = \mathbf{b}/\mu\cdot L_\mathbf{v}\mathbf{b}\ \Delta\Omega = \upsilon\mathbf{h}(\mathbf{b})\cdot L_\mathbf{v}\mathbf{b}\ \Delta\Omega.$$

The second term is the variation of energy associated with the variation of the function itself (independenlty of the variation of its argument), for geometrical reasons related with the fact that the Lie derivative also applies to the dot product operator $\cdot$ and to the volume element $\Delta\Omega$. The second term is the power $\dot{W}_{EM}$ delivered by magnetic forces, which, as shown in [2], [3], can be written as the tensor product of the gradient of the velocity field $\mathbf{v}$ and a tensor σ_{EM}, which we shall call the electromagnetic stress tensor. The electromagnetic stress tensor is not commonly invoked as such when dealing with magnetic forces, although it is the fundamental quantity representing the electromechanical coupling. The magnetic force density is the divergence of it, $\rho^F_{EM} = \operatorname{div}\sigma_{EM}$ and, at material interfaces where σ_{EM} is discontinuous, the divergence must be understood in the sense of distribution, i.e. as the jump of its normal component across the discontinuity.

Each material has its own expression of σ_{EM}. One has

$$\begin{aligned} \sigma_{EM} &= \frac{\mathbf{b}\mathbf{b}^T}{\mu} - \frac{|\mathbf{b}|^2}{2\mu}\mathbb{I} \\ \sigma_{EM} &= \frac{\mathbf{b}\mathbf{b}^T}{\mu(|\mathbf{b}|)} - \left(\frac{|\mathbf{b}|^2}{\mu(|\mathbf{b}|)} - \varrho^\Psi\right)\mathbb{I}, \quad \varrho^\Psi = \int_0^{|\mathbf{b}|}\frac{x}{\mu(x)}\,\mathrm{d}x \\ \sigma_{EM} &= \mu\mathbf{h}(\mathbf{b})\mathbf{h}^T(\mathbf{b}) - \mu\frac{|\mathbf{h}(\mathbf{b})|^2}{2}\mathbb{I}, \quad \mathbf{h}(\mathbf{b}) = \frac{1}{\mu}(\mathbf{b}-\mathbf{J}) \end{aligned}$$

with $\mathbb{I}$ the identity tensor, respectively for linear materials, isotropic saturable materials and permanent magnet materials. In particular, in empty space, $\mu = \mu_0$ and σ_{EM} is the classical Maxwell stress tensor.

III. Eggshell method

The Eggshell method [3] is the generic FE implementation of this principle. The Maxwell Stress Tensor (MST) method [4] and the Virtual Work Principle (VWP) method [5] are indeed particular cases of (2) with, for each, a specific choices of the velocity field $\mathbf{v}$. The nodal force F_N at node N of a mesh (Coulomb's method) is obtained by taking $\mathbf{v} \equiv \gamma_N\delta\mathbf{u}$ where

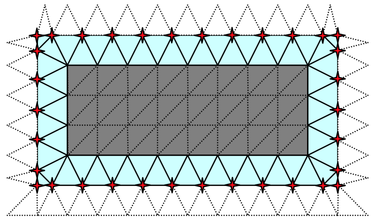

Fig. 1. User-defined eggshell in 2D

the $\delta\mathbf{u}$ is a vector and γ_N the nodal shape function of node N. One has then, by identification of $\dot{W}_{EM}$ with the work done by F_N,

$$\dot{W}_{EM} = \int_\Omega \sigma_{EM} \cdot \operatorname{grad}\gamma \;\; \mathrm{d}\Omega \cdot \delta\mathbf{u} \equiv \mathbf{F}_N \cdot \delta\mathbf{u}, \qquad (3)$$

and hence a definition for F_N after elimination of the arbitrary $\delta\mathbf{u}$. These nodal forces are the r.h.s. magnetic terms of the structural problem. Similarly, the resultant magnetic force $\mathbf{F}_Y$ acting on a rigid-body region Y is obtained by taking $\mathbf{v} \equiv \delta\mathbf{u}\;\gamma_Y$, where γ is any function equal to one on Y and decaying smoothly down to zero in the air region outside Y. One has thus, for both cases, a unique generic expression

$$\mathbf{F}_X = \int_\Omega \sigma_{EM} \cdot \operatorname{grad}\gamma_X \;\; \mathrm{d}\Omega, \qquad (4)$$

where the calculated magnetic force F_X is the one corresponding to the velocity field defined by the scalar function γ_X.

IV. FE implementation

Having recognized the common origin of the MST and VWP methods is an advantage when it comes to the FE implementation, which can then be done generically for 2D/3D problems and arbitrary materials. The only tool needed is the ability to define a continuous scalar FE field γ_X whose value is one at all nodes of a given region $X \in \Omega$ (the region X is pointwise, i.e. a single node, in case of Coulomb's method), and zero at all other nodes of Ω, and to evaluate its gradient. This gradient field vanishes everywhere except on a one element thick layer around X called *eggshell*. An efficient implementation requires to identify the elements of the eggshell (elements of $\Omega - X$ having at least one node in common with X), and to limit the integration (4) to this support. GetDP [6] can do this automatically in 2D and 3D. If this feature is not available however, the shell can also be defined by the user in the model's geometry, Fig. 1. For a generic material independent implementation, the code that evaluates (4) should also be organized so as to have σ_{EM} as a tensor parameter defined regionwise by the user.

V. Experimental validation

The experimental valiadtion setup consists of a fixed part holding the first magnet, and a mobile part holding the second magnet and to which the force sensor is fixed, Fig. 2. The distance between the magnets is adjustable by means of an accurate positioning vertical stage. The air-gap length ranges between 0 to 5mm and is monitored by a laser displacement sensor. A precision force sensor (0-200N) is used for measuring the magnetic force. The magnet dimensions (height $\times$ diameter) are 40 $\times$ 14mm for the fixed magnet and 10 $\times$ 14mm for the moving magnet. The PMs are sintered NdFeB (NEOFLUX-GSN35) with a remnant magnetic flux density of respectively $B_r = 1.24$T and 1.04T. Their relative magnetic permeability is $\mu_r = 1$. Fig. 3 shows the comparison of the measured forces with 3D scalar potential, 3D vector potential and 2D axisymmetric FE simulations. 3D simulations were done with code_Carmel [7], and 2D simulations with GetDP [6].

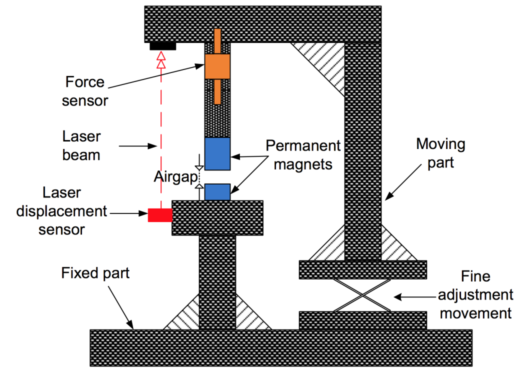

Fig. 2. Experimental setup

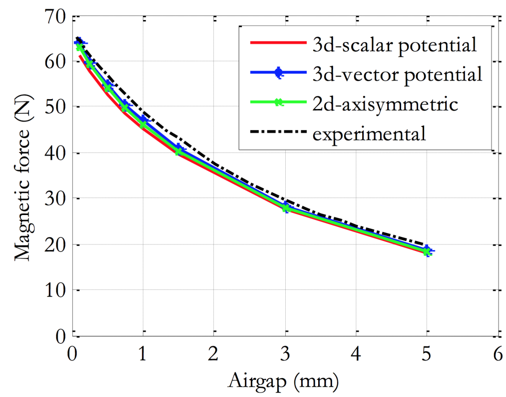

Fig. 3. Magnetic force as a function of the air-gap.

VI. Accuracy and convergence

Different aspects regarding the accuracy of the computed forces will be analyzed in the full paper: the position of the shell in the gap (contact with material objects or not), the thickness of the shell (aspect ratio of the elements), the meshsize in the shell, the position of the infinite boundary, etc.

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